

# **An Estimation Method for Spatiotemporal Traffic States Based on Incomplete Traffic Observation**

Ryuichi TANI, and Kenetsu UCHIDA  
Hokkaido University, Japan.

The 8th International Symposium on Dynamic Traffic Assignment,  
June 28 to June 30, 2021.

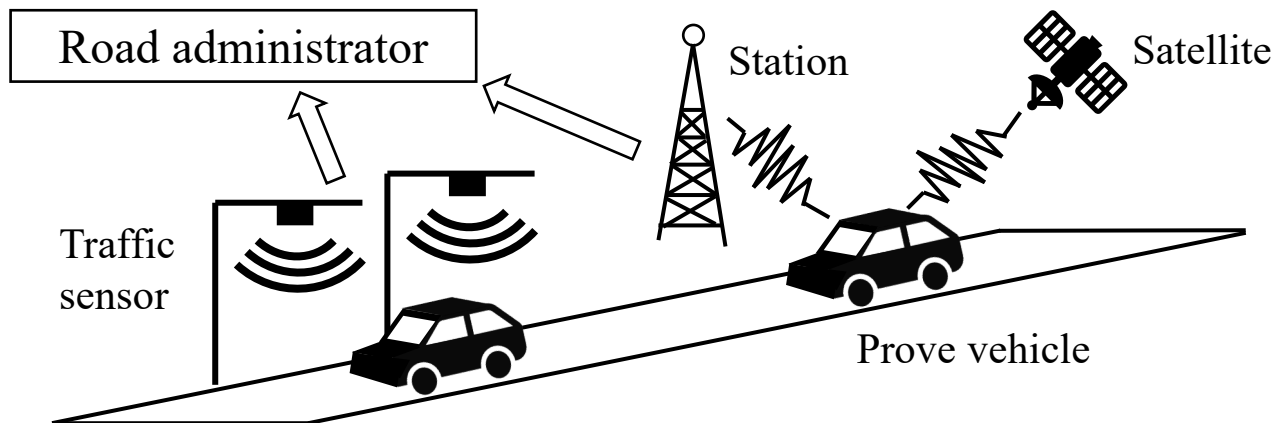


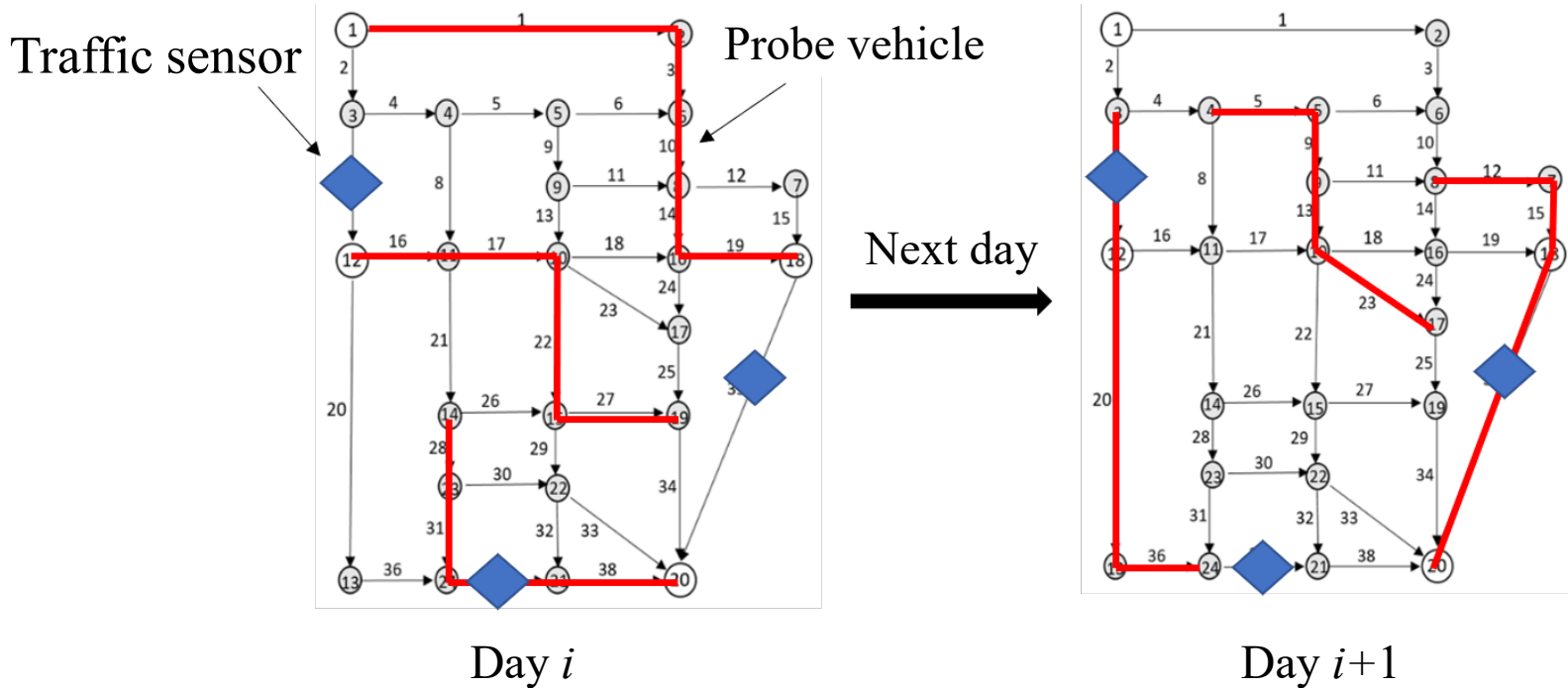
Network-based traffic state estimation with traffic sensor and probe car data

If the data observation is incomplete...

- low penetration of probe vehicles,
- privacy protection,
- less installation of traffic sensors

**Impossible to collect perfect data set spatiotemporally**



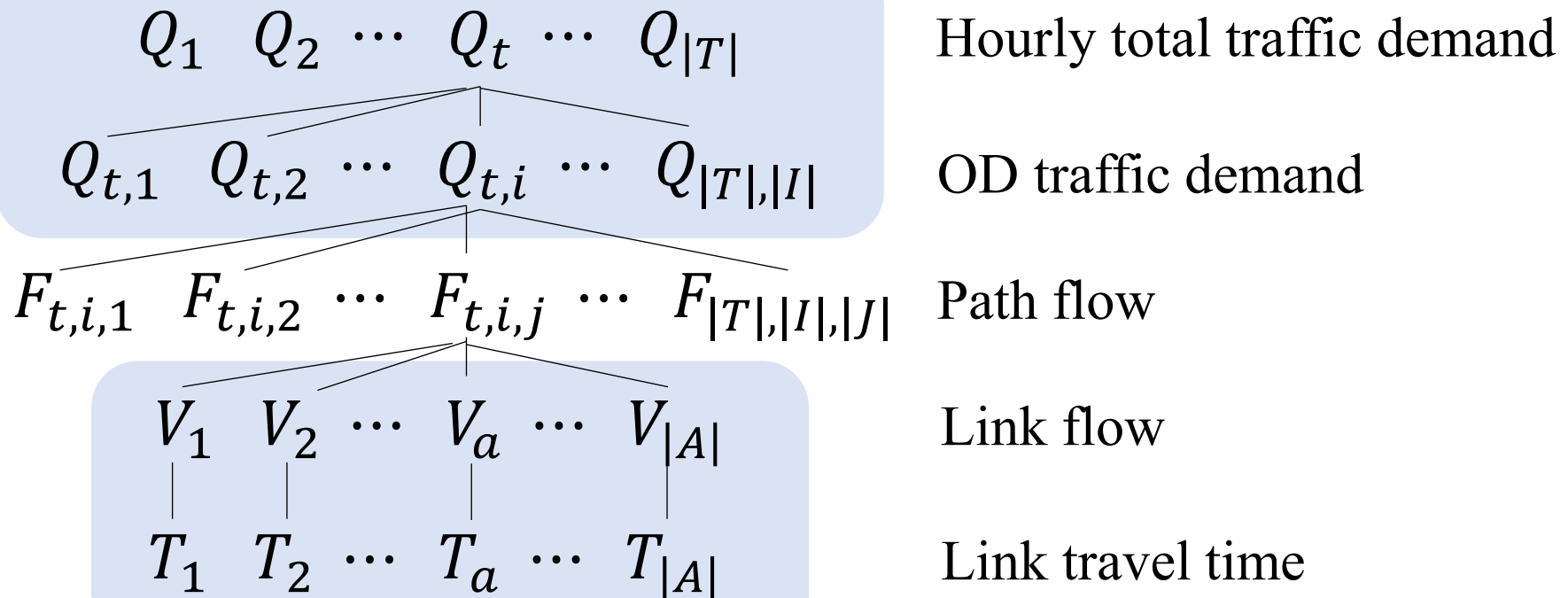


- Day-to-day observation
- The number of traffic sensors and the scale of traffic demand are comparatively small than network size



- to estimate traffic states as network-sized multivariate random variables
- to consider the missing data of the traffic observation

## Unknown valuables



## Observed valuables



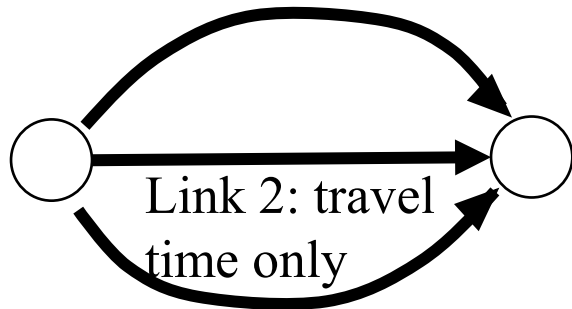
## Estimation target

Link flow :  $\mathbf{V} \sim MVLN(\boldsymbol{\mu}_V, \boldsymbol{\Sigma}_V)$

Link travel time (delay time):  $\mathbf{T} \sim MVLN(\boldsymbol{\mu}_T, \boldsymbol{\Sigma}_T)$

## Data missing

Link 1 : flow and travel time



Link 3: no observation

## Observed data set

$$\mathbf{d}_g = (\hat{v}_1, \hat{t}_1, -, \hat{t}_2, -, -) \quad \forall g \in \{1, \dots, G\}$$

## Shrink the dimension

$$\mathbf{M}_g = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad \forall g \in \{1, \dots, G\}$$

- Each data set can be missed
- **Likelihood estimation considering data missing**



## Full information maximum likelihood function

$$L(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \prod_{g=1}^G \frac{\exp\left(-\frac{1}{2}(\widehat{\mathbf{d}}_g - \boldsymbol{\mu}_g)^T \boldsymbol{\Sigma}_g^{-1}(\widehat{\mathbf{d}}_g - \boldsymbol{\mu}_g)\right)}{2\pi^{n(g)/2} |\boldsymbol{\Sigma}_g|^{1/2}}$$

where

$$\boldsymbol{\Sigma}_g = \mathbf{M}_g \boldsymbol{\Sigma} \mathbf{M}_g^T, \quad \boldsymbol{\mu}_g = \mathbf{M}_g \boldsymbol{\mu}^T, \quad \widehat{\mathbf{d}}_g = \mathbf{M}_g \mathbf{d}_g^T$$

With  
equilibrium  
constrain

## Traffic assignment considering network uncertainty

$$E[F_{ij}] = p_{ij}(\mathbf{c}(\mathbf{F})) \cdot E[Q_i] \quad \forall i \in \mathbf{I}, \forall j \in \mathbf{J}_i$$

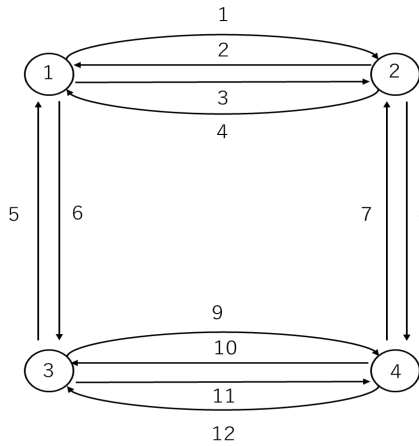
where

$$p_{ij} = \frac{\exp(-\phi \cdot c_{ij}(\mathbf{F}))}{\sum_{j=1}^{|\mathbf{J}_i|} \exp(-\phi \cdot c_{ij}(\mathbf{F}))} \quad \forall i \in \mathbf{I}, \forall j \in \mathbf{J}_i$$

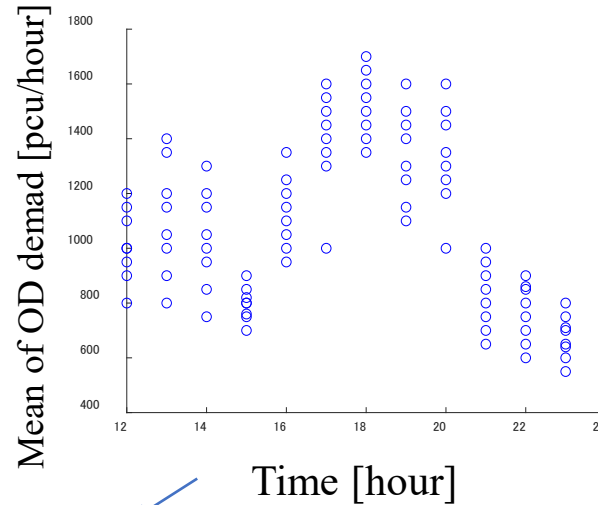
Fixed point problem  
Logit-type SUE

$$\mathbf{c} = \left( c_{11}(\mathbf{F}), \dots, c_{i|\mathbf{I}| \cdot |\mathbf{J}_{|\mathbf{I}|}}(\mathbf{F}) \right), \quad \mathbf{F} = \left( F_{11}, \dots, F_{|\mathbf{I}| \cdot |\mathbf{J}_{|\mathbf{I}|}} \right)$$





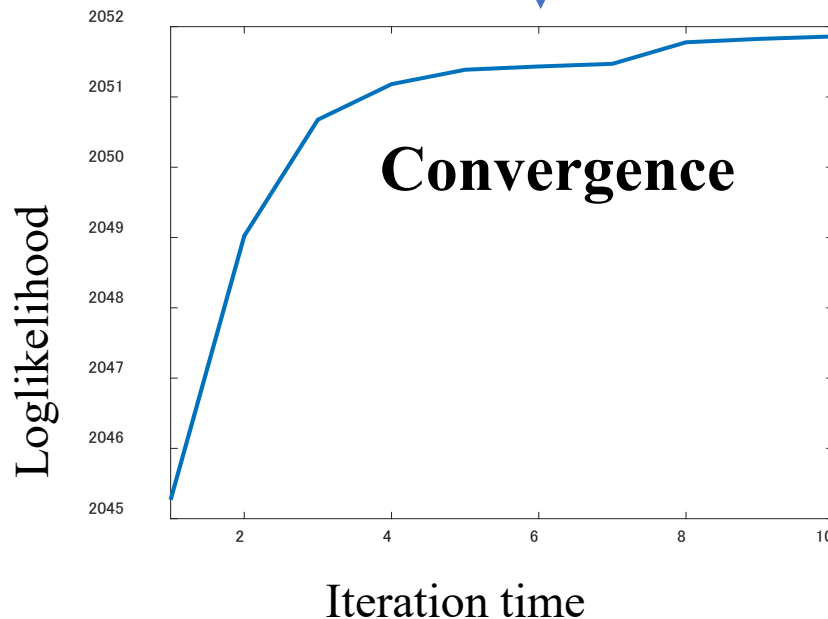
Test network



## Major settings

- 12 time intervals
- Probe ratio: 30%
- Sample size: 300

## Traffic state estimation



- Solved two auxiliary problems iteratively
- Estimated mean and variance-covariance of traffic demands



- 
- Network-based traffic states estimation for several time intervals
  - With missing spatiotemporal data
    - traffic sensor; probe vehicle
  - Numerical test showed the solution conversion

## Next step

- Realtime prediction of the traffic states from the latest observation
  - Bayesian estimation considering data missing

